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Ionization Rates for Atmospheric and Ionospheric Breakdown

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The differences in the ionization rates for RF induced atmospheric breakdown between the analytic formula given by *Gurevich et al.* [1978] and recent numerical simulations *Short et al.*, 1991; *Tsang et al.*, [1991] were examined and clarified. An updated analytic formula for the ionization rate of air, which includes corrections due to kinetic effects of the electron distribution function, was derived. The formula is valid for arbitrary ratios of RF to electron-neutral collision frequency.

1. Ionization Rates

Theoretical and experimental studies of air breakdown in the presence of strong microwave fields were performed as early as the late 1950s and early 1960s [*McDonald*, 1966]. These studies emphasized breakdown under controlled laboratory conditions mostly in microwave cavities. *Kroll and Watson* [1972] extended the microwave breakdown computations to optical and infrared laser frequencies including the effects of multiphoton absorption. Interest on the subject has been recently renewed, since the development of microwave sources and phased arrays provides the technological capability to create plasma clouds in the atmosphere and ionosphere, using focused ground-based installations, which transmit short high-power pulses of microwaves [*Gurevich*, 1980]. These plasma clouds can be used as artificially ionized mirrors (AIM) for communications and radar applications [*Borisov et al.*, 1986; *Armstrong et al.*, 1989; *Duncan and Milikh*, 1989; *Short et al.*, 1990; *Tsang et al.*, 1991]. Recent studies [*Papadopoulos*, 1990] indicate that optical and other emissions from such plasma clouds can be used to diagnose remotely minority and toxic species in the stratosphere.

A critical quantity in assessing the RF power requirements for pulsed breakdown experiments is an accurate knowledge of the ionization rate ν_i and its dependence on the local RF electric field E_o, frequency ω, and ambient air neutral density N_m or pressure P. Empirical formulae for ν_i have been presented by several authors [*Ali*, 1981; *Lupan*, 1976; *Mayhan and DeVore*, 1968]. These formulae are, however, valid for limited ranges of parameters, often contradictory and without solid physical justification [*Short et al.*, 1990; *Borisov et al.*, 1986]. The consistent approach, which is valid for any meaning of the radio wave frequency is associated with the kinetic theory of breakdown.

An analytic theory for the ionization rate was developed by *Gurevich et al.* [1978] (hereafter referred to as I). An important contribution of the paper was the recognition that ν_i is a function of two dimensionless parameters E_o/E_k and ω/ν_k. The value of the effective collision frequency ν_k is only a function of the ambient gas density given by

$$\nu_k = 1.6 \times 10^{-7}\left(\frac{N_m}{cm^3}\right)s^{-1} = 6 \times 10^9\left(\frac{P}{Torr}\right)s^{-1} \tag{1}$$

The value of the characteristic field E_k depends in addition to the value of N_m, on the frequency ω in the form of ω/ν_k. It is defined as

$$E_k = 32\left(\frac{N_m}{2.7 \times 10^{19} cm^{-3}}\right)\sqrt{1+\frac{\omega^2}{\nu_k^2}} = 32\left(\frac{P}{760\ Torr}\right)\sqrt{1+\frac{\omega^2}{\nu_k^2}}\ \frac{kV}{cm} \tag{2}$$

The ionization rate was given in I in the form

$$\frac{\nu_i}{\nu_m} = C_1 F\left(\frac{E_o}{E_k}\right)\left(\frac{E_o}{E_k}\right)^2 \exp\left[-C_2\left(\frac{E_k}{E_o}\right)\right] \tag{3}$$

In (3), ν_m is given by

$$\nu_m = 7.6 \times 10^{-13}\left(\frac{N_m}{cm^3}\right)s^{-1} \tag{4}$$

and the function $F(x)$ is defined as

$$F(x) \equiv \frac{1 + 6.3\exp(-2.6/x)}{1.5} \tag{5}$$

The two terms of F(x) reflect the differences in the ionization properties of N_2 and O_2, the dominant atmospheric components taken as 80% and 20% of the neutral density. For values of $E_o/E_k \approx 1-5$ relevant to ionospheric breakdown the factor F varies between 1 and 3.

The parameters C_1 and C_2 in (3) are parametrizations of the integrals of inelastic cross sections over the energy. They depend on matching the electron distribution function in energy regions where different inelastic losses dominate (see I for details). In the general case, C_1 and C_2 are functions of the wave frequency. However, in I it was assumed that C_1 and C_2 are constant with values

$$C_1 = 5.5 \times e^6,\quad C_2 = 6 \tag{6}$$

By using (5) and (6), the ionization rate ν_i^I may be extrapolated:

$$\frac{\nu_i^I}{\nu_m} = (5.5 \times e^6)\, F\left(\frac{E_o}{E_k}\right)\left(\frac{E_o}{E_k}\right)^2 \exp\left[-6\frac{E_o}{E_k}\right] \tag{7a}$$

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This equation can be represented in the form

$$\frac{\nu_i^I}{\nu_m} = 5.5F\left(\frac{E_o}{E_k}\right)\left(\frac{E_o}{E_k}\right)^2 \exp\left\{-6\left(\frac{E_o}{E_k}-1\right)\right\} \quad (7b)$$

A set of computer simulations of ionospheric breakdown performed recently [*Short et al.*, 1990; *Tsang et al.*, 1991] (hereafter referred to II), while confirming the general dependence of ν_i on E_o/E_k and ω/ν_k, indicated that (7) significantly overestimates the value of ν_i and consequently underestimates the required power for pulsed breakdown. In the high-frequency regime ($\omega >> \nu_k$) the simulation results were consistent with the analytic expression [*Papadopoulos*, 1990], (hereafter referred to as III):

$$\nu_i^{III}/\nu_m = 1.2 \times 10^3 \left(\frac{\tilde{\epsilon}}{eV}\right)\exp\left(-\sqrt{\frac{2eV}{\tilde{\epsilon}}}\right) \quad (8a)$$

where $\tilde{\epsilon}$ is the electron quiver energy in electron volts defined as

$$\tilde{\epsilon} = \frac{1}{2}m\frac{e^2E_o^2}{m^2\omega^2} \quad (8b)$$

In the $\omega >> \nu_k$ range the value of $\tilde{\epsilon}$ corresponding to E_k is $\tilde{\epsilon}_k = .08eV$.

A comparison of (7a) and (7b) with (8a) and (8b) along with the results of the numerical simulations [*Short et al.*, 1990; *Tsang et al.*, 1991] for $\omega >> \nu_k$ is shown in Figure 1 (see also Table 1). Equation (8) gives results consistent with the simulation up to $E_o/E_k \approx 5$. It can be seen also that (7) overestimates the ionization rate by more than an order of magnitude over the entire range. This statement is backed up by the comparison of the simulation results with the collection of experimental data adopted from *Sharfman and Morita* [1964]. When the values of E_e/p, V/cm Torr used in the mentioned paper, where transformed into the units of E_o/E_k, we assumed that $\omega/\nu_k < 1$.

This overestimate has important implications in the microwave power requirements in performing atmospheric breakdown experiments as can be seen from Table 2. Planning based on (7) requires a value of electric field smaller by a factor of 1.5 - 1.8 in the breakdown region than the numerical results. This is translated into an underestimate of the microwave power by a factor of 3.

In the next section we discuss briefly the causes of the discrepancy and determine the values of the parameters C_1 and C_2 as a function of ω/ν_k, which are required to reconcile the results of reference I with these of reference II.

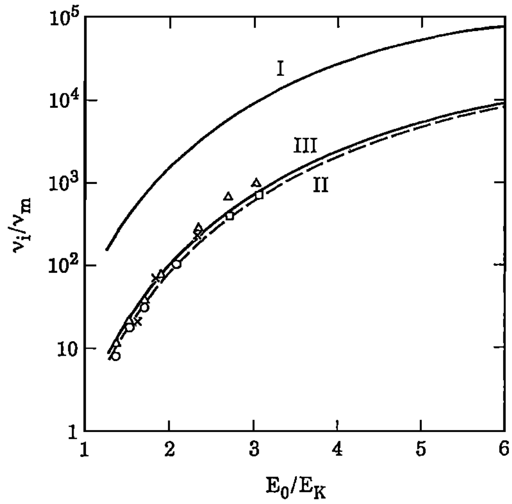

Fig. 1. The ionization frequency ν_i normalized over ν_m as obtained in high frequency regime ($\omega >> \nu_k$) for different amplitude of the electric field of microwave. Curves I and III correspond to the analytical theory referred as I and III, while curve II corresponds to computer simulation, referred as II. Shown by open circles, open triangles, crosses, and boxes is the collection of the different experimental data adopted from *Sharfman and Morita* [1964, Figure 1].

TABLE 1. Ionization Frequency ν_i Normalized Over ν_m, Which Is Obtained for Different Amplitude of the Electric Field of Microwave ($\omega >> \nu_k$)

	$E_o(\frac{kV}{m}) \times 10^4$						
	0.61	0.87	1.1	1.48	1.5	1.9	2.4
E_o/E_k	1.32	1.86	2.28	3.19	3.23	4.07	5.13
$\frac{\nu_i^I}{\nu_m}/10^3$	0.2	1.0	2.7	10.7	11.3	26.0	56.3
$\frac{\nu_i^{II}}{\nu_m}$	8.1	72	210	930	970	2,250	4,650
$\frac{\nu_i^{III}}{\nu_m}$	7.9	66	190	810	860	2,040	4,490
$\frac{\nu_i^{(9)}}{\nu_m}$	7.2	60	180	840	880	2,210	5,060

I, III corespond to the analytical theory refered as I, III; II - to the computer simulation. Frequencies obtained by formula (9) are indicated as $\nu_i^{(9)}$.

2. Updated Ionization Rates: Kinetic Corrections

The difference in the ionization frequency between reference I and II can be attributed to several factors. First, it is attributed to differences in the inelastic collision cross sections that were used. Reference I used cross sections adapted from *Frost and Phelps* [1962], and *Hake and Phelps* [1967], while reference II used updated values based on recent measurements [*Ali*, 1981]. Second, the approximation used by the analytic theory in reference I, equivalent to the Born approximation, is not sufficiently accurate over the entire range. Finally, the parameters C_1 and C_2 in (3) are not constant, as was assumed in reference I, but are functions of ω/ν_k.

In the remainder of this section we utilize the self-similar form for the ionization rate as a function of E_o/E_k and ω/ν_k given by (3), in conjunction with the computer code used in reference II, to derive a new accurate analytic formula which describes the ionization rate for the entire range of ω/ν_k. To accomplish this, we assume ν_i in the form

$$\frac{\nu_i}{\nu_m} = F\left(\frac{E_o}{E_k}\right)\left(\frac{E_o}{E_k}\right)^2 \times C_1'\left(\frac{\omega}{\nu_k}\right)\exp\left[-4.7\left(C_2'\left(\frac{\omega}{\nu_k}\right)\frac{E_k}{E_o}-1\right)\right] \quad (9)$$

The coefficient 4.7 appears in the exponent of (9) instead of coefficient 6 given in (7), to account for the use of updated inelastic cross sections. The parameters $C_1'(\omega/\nu_k)$ and $C_2'(\omega/\nu_k)$ will be determined numerically. They are essentially kinetic coefficients. To determine these coefficients, a series of numerical simulations were performed, to compute the ionization rate as a function of $x = E_o/E_k$ for fixed values of ω/ν_k. For each value of ω/ν_k the values of C_1' and C_2' were determined from the relationship

TABLE 2. Electric Field Required to Produce Corresponding Ionization Frequency ν_i/ν_m in High-Frequency Regime ($\omega >> \nu_k$)

	ν_i/ν_m		
	30	300	1000
$\frac{E_o^I}{E_k}$	1.1	1.44	1.83
$\frac{E_o^{II}}{E_k}$	1.65	2.5	3.25

E_o^I/E_k and E_o^{II}/E_k are calculated by the different methods described in references I and II correspondingly.

$$f = \ln\left[\frac{\nu_i(x)}{\nu_m}\frac{1}{x^2F(x)}\right] = 4.7 + \ln C_1' - \frac{C_2'}{x} \tag{10}$$

shown in Figure 2.

The linear part of this function defines the constant $C_2'(\omega/\nu_k)$, while the cutoff value f_o at the ordinate axis is $4.7 + \ln C_1'$, i.e. $C_1' = \exp(f_o - 4.7)$. Figure 3 shows the functions C_1' and C_2'. It can be seen that under $\omega << \nu_k$, both C_1' and C_2' tend to 1.0. In the opposite case $\omega >> \nu_k$, $C_1' \rightarrow 1.5$, $C_2' \rightarrow 1.1$.

As can be seen from Table 1, in the high-frequency regime $\omega >> \nu_k$ (9) with $C_1' = 1.5$ and $C_2' = 1.1$ reduces to the analytic formula of reference III, given by (8) and is, of course, consistent with the results of the computer simulation.

In general, the analytic formula given by (9) with the corresponding kinetic coefficients $C_1'(\omega/\nu_k)$, $C_2'(\omega/\nu_k)$ presents values of the ionization frequency ν_i with an the accuracy better than 10% for the amplitude of electric field close to the threshold ($E_o/E_k \leq 3$). If the electric field increases up to 5 E_k the accuracy of ionization frequency obtained by formula (9) would be no better than 20–25%. For the fields above 5 E_k the simple kinetic approach developed in references I-III can be no longer applicable, since under such conditions the mean electron energy tends to the ionization energy which makes the ionization process nonstationary. The mean source of energy losses becomes associated with the ionization and heating of the newly born electrons. It changes the form of the Fokker-Plank equation and effects the electron distribution function. This requires different analysis [*Borisov et al.*, 1986].

3. Ionization Threshold

It is customary in the literature to refer to a breakdown power or electric field threshold. The definition of the threshold depends critically on the dominant losses occurring in the particular experiment and pulse length of the microwaves. The lowest possible threshold occurs if we assume asymptotically long irradiation time and neglect convective or diffusive losses. In this case a unique definition of the threshold is possible as the power density (or electric field) at which the electron losses due to dissociative attachment of O_2 equal the ionization rate. The threshold electric field E_{th} as defined above can be found by equating the ionization rate to the attachment rate. The values of E_{th} were measured in dc field [see *Kroll and Watson*, 1972]. For the high-frequency microwaves the E_{th} was obtained in reference II. Using computer simulation of the same type as in reference II, we can find that

$$E_{th}\left(\frac{\omega}{\nu_k}\right) = C_{th}'\left(\frac{\omega}{\nu_k}\right)E_k \tag{11}$$

where the coefficient C_{th}' is 1.0 when $\omega/\nu_k << 1$ and becomes equal to 1.6 when $\omega/\nu_k >> 1$.

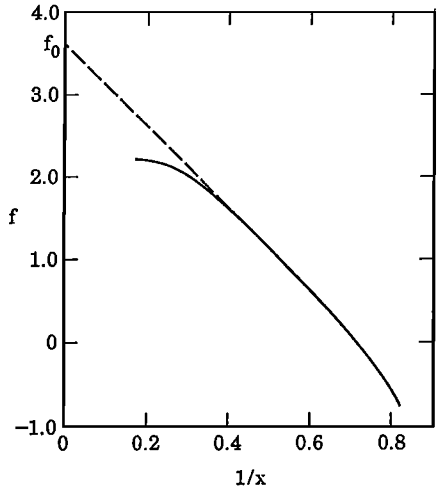

Fig. 2. Illustration of the method of determination of kinetic coefficients, which is based on the relationship (10).

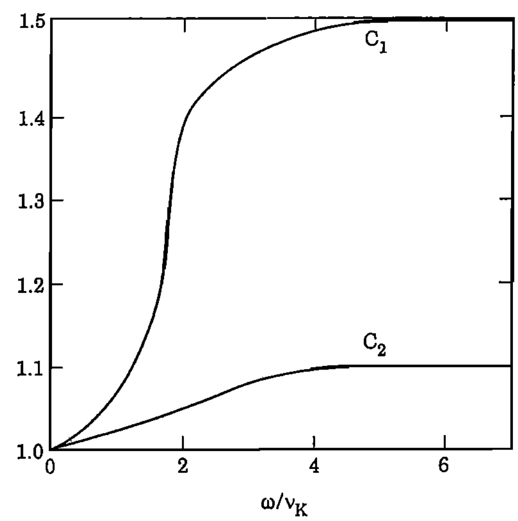

Fig. 3. The kinetic coefficients C_1' and C_2' as a functions of ω/ν_k.

4. Summary and Conclusions

The reasons for the differences in the ionization rates between reference I and references II and III were discussed. Using the self-similar functional form for the ionization rate derived in reference I in conjunction with computer simulations, an analytic form for the ionization rate as a function of the RF frequency and power was derived, which is valid for the entire collisionality range (that is, $\omega/\nu_k \lessgtr 1$) and for $E_o/E_k < 5$.

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